

On Force-free Eigenstates for Ball Lightning

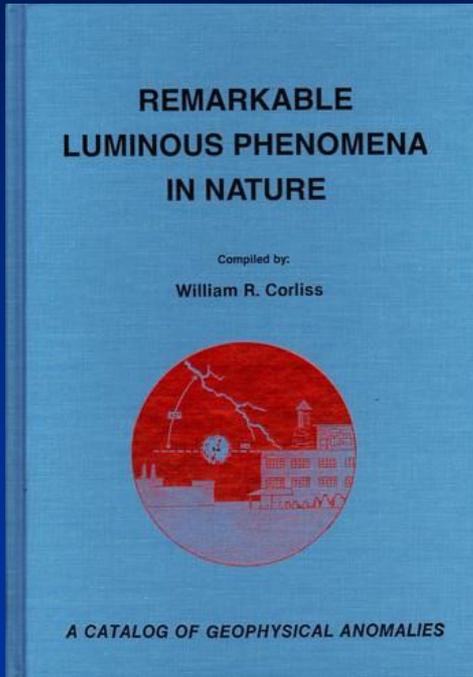
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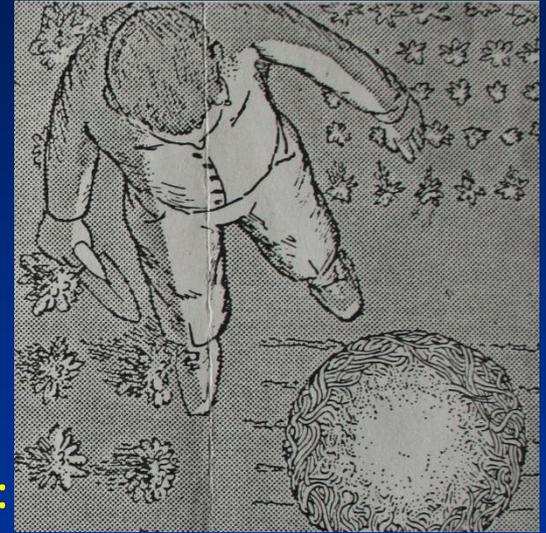
Video presentation at 3rd International Symposium on Lightning and Storm Related Phenomena (ISL-SRP-III). Aurillac, France, 06-08 October 2021. © G.C.Dijkhuis 2021

Ball Lightning with Internal String Structure



Case 1: England, 1887.
The eye witness walks in heavy thunderstorm. Silent formation and decay, size of a cricket ball. Apparent fibres of light are “wriggling like a mass of eels”.

Case 2: Coventry, UK, 1940.
Eye witness E. Matts in his garden: no rain or thunder, size 2 ft across, pale blue-green colour, 3 seconds, explosive decay damages house.



Fibers of light wriggle on surface of ball lightning

Case 1: “a vibrating mass of strips of light, in rapid rolling motion”

Case 2: “the ball seemed made of a mass of writhing strings of light”

Ball Lightning inside a Jet Plane

Eye witness M. Newroth was underway from Las Vegas to St. Paul, USA. Flying between two storms the plane was racing to avoid bad weather.

Aggressive St. Elmo's fire on central window pane triggered a ball lightning



Dassault Falcon 20



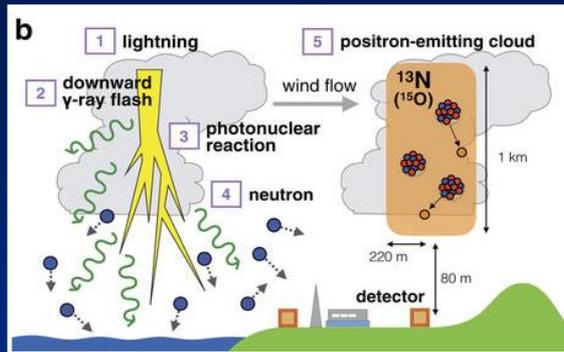
Cockpit



Rear view

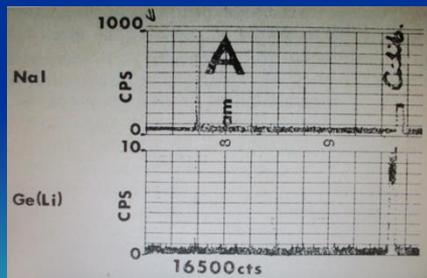
Newroth's 2006 *perfect ball* dovetails nicely with Jennison's iconic *perfect sphere* floating down the aisle of a commercial airliner four decades earlier.

Nuclear Activity in Thundercloud

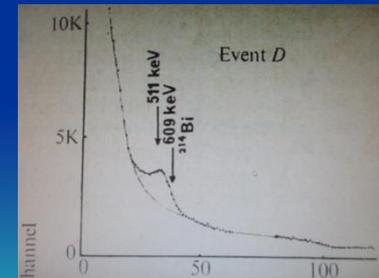


Kashiwazaki-Kariwa, Japan
Lightning from a winter thunderstorm struck near a monitor station. Detectors recorded γ -radiation at 0.511 MeV as from electron-positron annihilation.
Source: T. Enoto, *Nature*, 2017

Gulmarg Cosmic Ray Station, India.
Operation at 2700 m in Himalaya since 1951.
Neutron counts from lightning. *Nature* 1985.
Fusion energy from 2.45 MeV neutrons.
Source: P.M. Ishtiaq, *J. Geophys. Res.*, 2016



UKAEA Culham, England
Atmospheric γ -radiation detected.
Peak at photon energy 511 keV recorded during thunderstorm.
Source: D.E. Ashby, *Nature* 1971



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Matrices for Electromagnetic Field and Potentials

Let electromagnetic vectors $\mathbf{E}=\{E_x, E_y, E_z\}$ and $\mathbf{B}=\{B_x, B_y, B_z\}$ define field matrix \mathbb{F} :

$$\mathbb{F} \equiv \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}, \quad \mathbb{f} \equiv \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix}, \quad \begin{aligned} \Delta\mathbb{F} &= (\mathbf{E} \cdot \mathbf{B})^2 \\ \Delta\mathbb{f} &= 0 \end{aligned}$$

Field determinant $\Delta\mathbb{F}$ is positive or zero, and its leading minor $\Delta\mathbb{f}$ is zero

Let matrix \mathbb{P} combine potentials Φ and \mathbf{A} defined by $\mathbf{E}=-\text{grad}\Phi$ and $\mathbf{B}=\text{rot}\mathbf{A}$:

$$\mathbb{P} \equiv \begin{bmatrix} \Phi & -A_x & -A_y & -A_z \\ A_x & \Phi & A_z & -A_y \\ A_y & -A_z & \Phi & A_x \\ A_z & A_y & -A_x & \Phi \end{bmatrix}, \quad \mathbb{p} \equiv \begin{bmatrix} \Phi & A_z & -A_y \\ -A_z & \Phi & A_x \\ A_y & -A_x & \Phi \end{bmatrix}, \quad \begin{aligned} \Delta\mathbb{P} &= (\Phi^2 + \mathbf{A} \cdot \mathbf{A})^2 \\ \Delta\mathbb{p} &= \Phi\sqrt{\Delta\mathbb{P}} \end{aligned}$$

Remarkably anti-symmetric pairs in \mathbb{P} precisely copy those in field matrix \mathbb{F} .

Chiral Plasma Dynamics

Partial derivatives of \mathbb{P} define matrices $\{u,i,j,k\}$ for quaternion algebra:

$$u \equiv \partial\mathbb{P}/\partial\Phi ; i \equiv \partial\mathbb{P}/\partial A_x ; j \equiv \partial\mathbb{P}/\partial A_y ; k \equiv \partial\mathbb{P}/\partial A_z$$

$$\mathbb{P} = u\Phi + iA_x + jA_y + kA_z ; -u = i^2 = j^2 = k^2 = i \cdot j \cdot k$$

Potentials Φ and \mathbf{A} define the field vectors $\mathbf{E} = -\text{grad}\Phi$ and $\mathbf{B} = \text{rot}\mathbf{A}$.

Quaternion calculus needs 12 relations between 16 partial derivatives:

Gradient	Rotation	Moment	Eigenvalue
$\partial\mathbf{A}/\partial w = -\text{grad}\Phi = \mathbf{E}$	$\text{rot}\mathbf{A} = \mathbf{B} = \mathbf{0}$	$\mathbf{x} \times \mathbf{A} = \mathbf{0}$	$\mathbb{P} \cdot \mathbf{x} = \mathbf{x} \partial\Phi/\partial w$

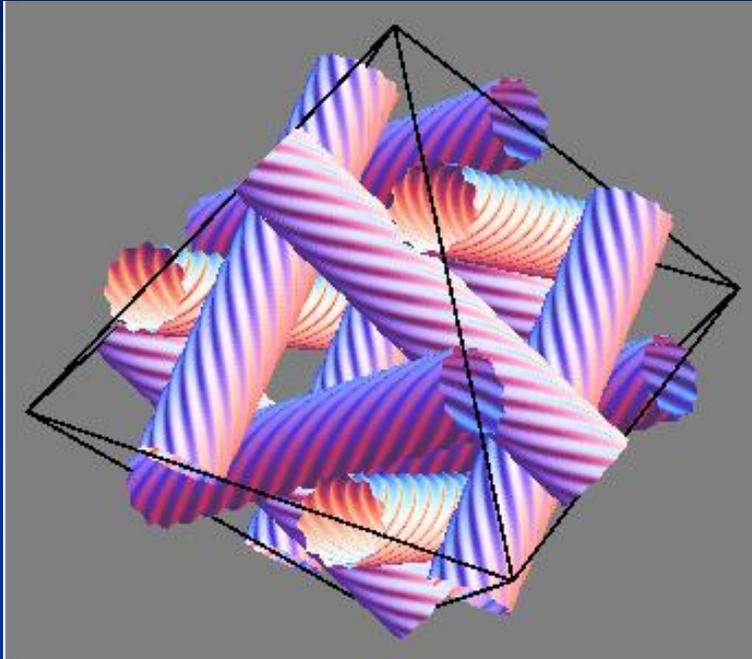
Projections return Cauchy-Riemann rules ; inductive proofs: ESA, Versailles, 2008.

From split form of potential matrix \mathbb{P} for real eigenvalues in chiral eigenspace :

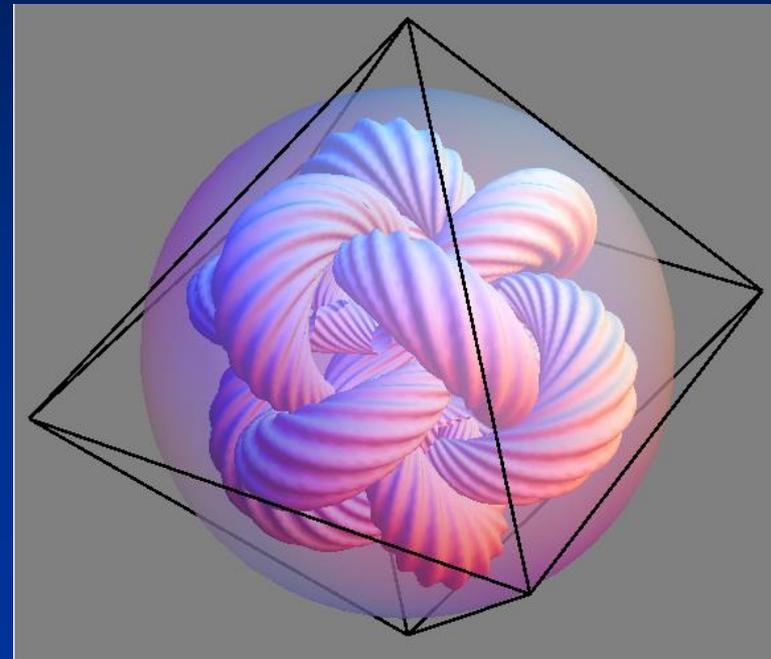
Eigenvalues	Eigenvector	Chirality
$\Phi \pm \Gamma$	$\{\mathbf{x} \ y - \Gamma z, -\Gamma y + z \ \mathbf{x}, 0, y^2 + z^2\}$	$\Gamma^2 \equiv -x^2 + y^2 + z^2$

Coherent Rotation of Chiral Tubes and Loops

Quaternion inversion map: $\{W, X, Y, Z\} = \frac{\{w, -x, -y, -z\}}{w^2 + x^2 + y^2 + z^2}$



Unit cell defines octahedral lattice of flux tubes with chiral surface.



The flaring image loops converge on the octahedral centerpoint.

Split-Quaternions have Real Eigenspace

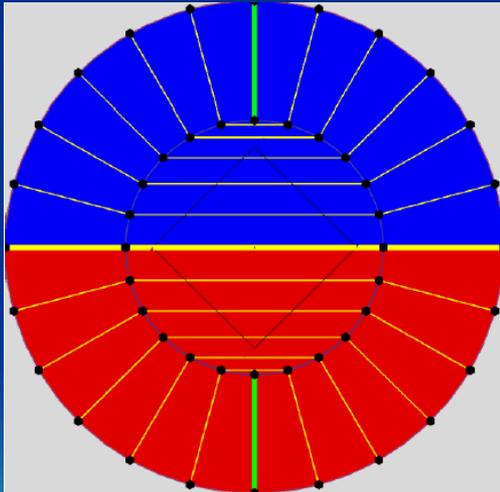
Real Γ for real eigenvectors:

$$\begin{aligned} & \{ \{ x y - \Gamma z, z x + \Gamma y, 0, y^2 + z^2 \}, \\ & \{ -z x - \Gamma y, x y - \Gamma z, y^2 + z^2, 0 \}, \\ & \{ x y + \Gamma z, z x - \Gamma y, 0, y^2 + z^2 \}, \\ & \{ -z x + \Gamma y, x y + \Gamma z, y^2 + z^2, 0 \} \} \end{aligned}$$

Sign reversals in split-calculus rules:

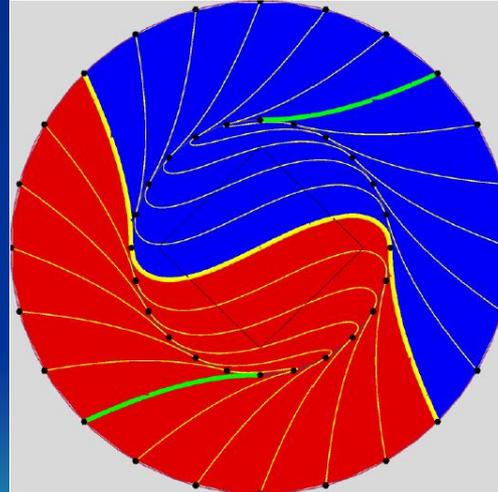
Split-Gradient	Split-Rotation
$\partial_x \Phi + \partial_w A_x = 0$	$\partial_z A_y - \partial_y A_z = 0$
$\partial_y \Phi - \partial_w A_y = 0$	$\partial_x A_z + \partial_z A_x = 0$
$\partial_z \Phi - \partial_w A_z = 0$	$\partial_y A_x + \partial_x A_y = 0$

Inner dots join flux lines



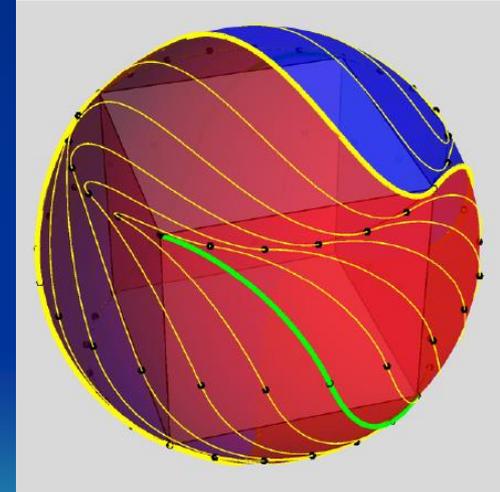
Straight field lines kink

Separatrix curve is white



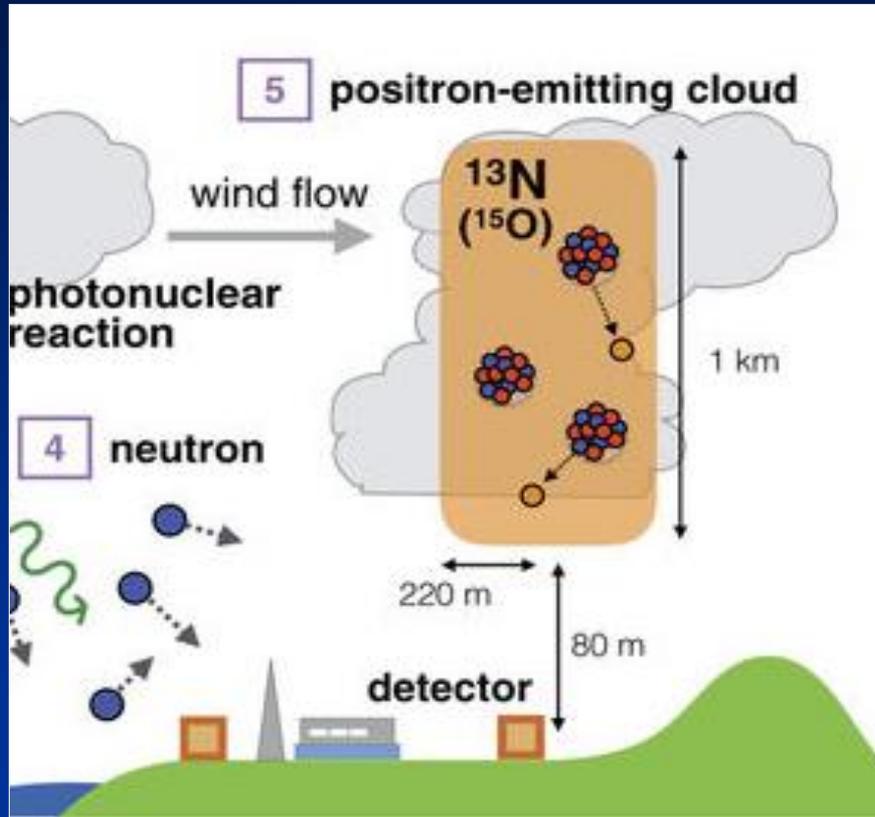
Smooth field lines swirl

Dipolar string is green



Smooth loops are nested

Radioactive Lightning and Laboratory Bosons



Nuclear process

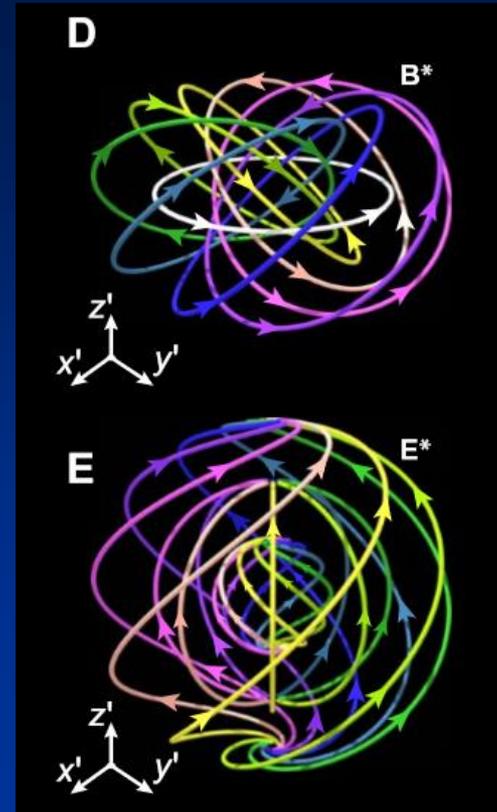


Source: T. Enoto et al., *Nature*, 2017

Photon energy

> 10 MeV

0.511 MeV



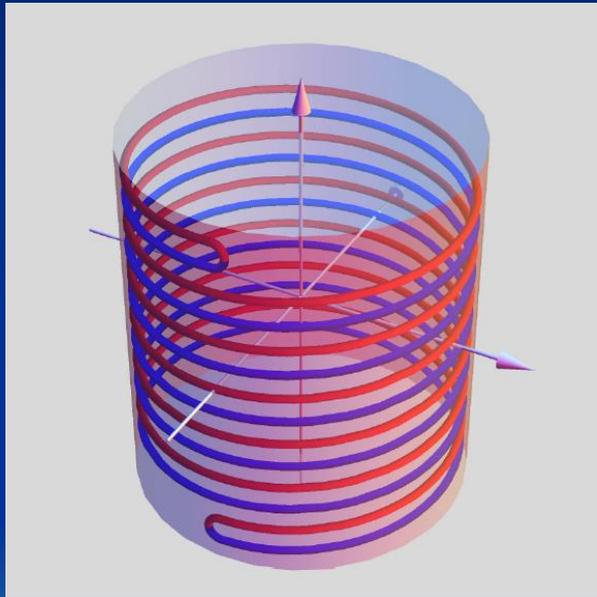
Skyrmion state links magnetic loops in (D), twists electric field lines in (E).

W. Lee et al., *Science Adv.*, 2018

Force-free Energy Storage in Chiral Flux Loops

In lightning channel:

- external torsion couple winds flux loop around discharge plasma,
- circulation at quantum limit scales energy linearly with loop length.

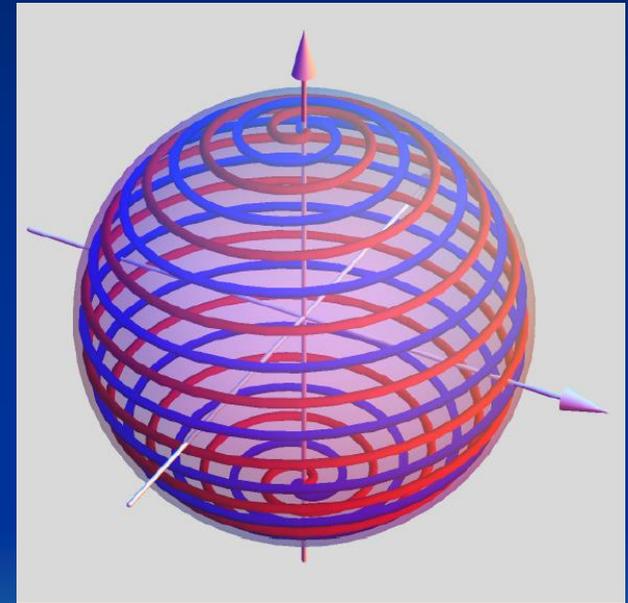


Double spiral aligns vorticity, current and magnetic field.

Pyroelectric respons by chiral symmetry

In ball lightning:

- radial pressure balance preserves shape and size of ball lightning,
- internal elastic force shrinks loop length sustaining radiative output.

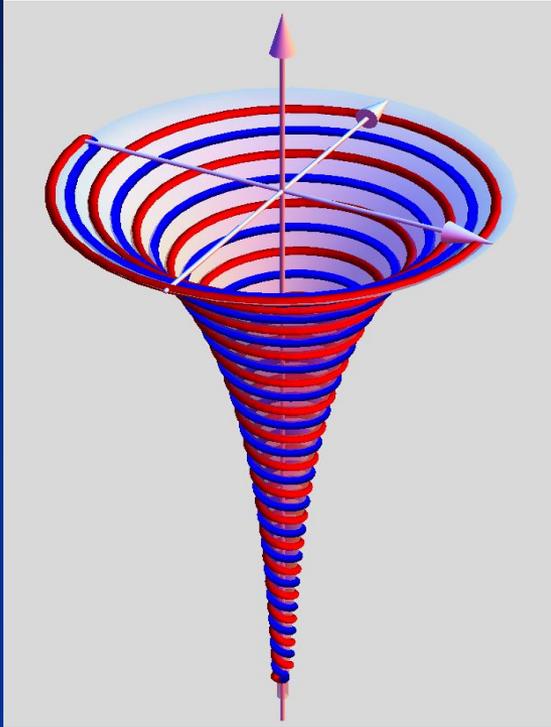


Red:
upward
current

Blue:
down-
ward
current

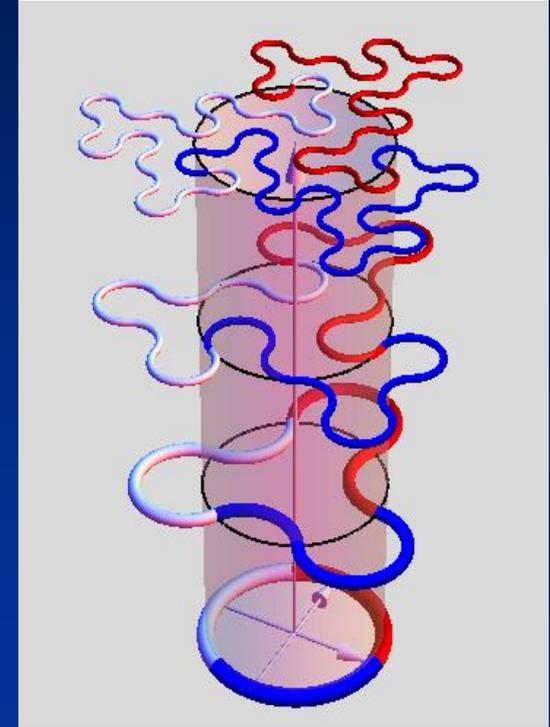
Canonical form by periodic coordinates

Particle Acceleration in Lightning Channel



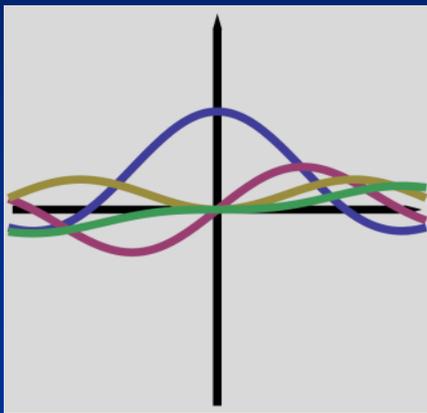
Lightning leader winds flux loop around channel axis.
< Peano process fits infinite loop length into finite area. Quantized circulation scales energy with loop length.

Identical arc units fill plane by successive scaling stages. Peano process fits infinite loop length into finite area. Quantized circulation scales energy with loop length. >

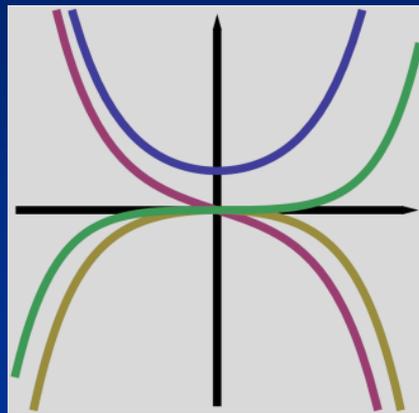


Bessel Functions in the Complex Plane

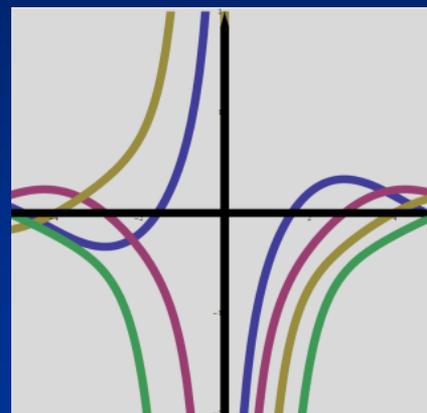
Zeros and divergencies show up in plots of the real and imaginary parts of spherical Bessel functions



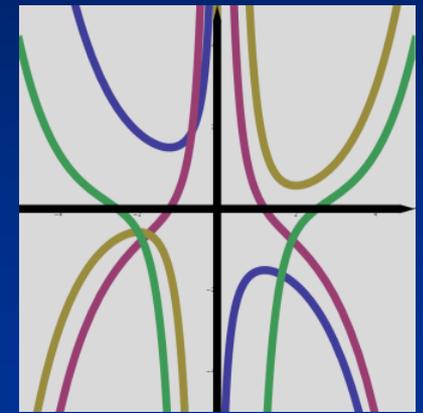
(a) $\text{Re}[j_n(w)]$



(b) $\text{Im}[j_n(x)]$



(c) $\text{Re}[y_n(w)]$



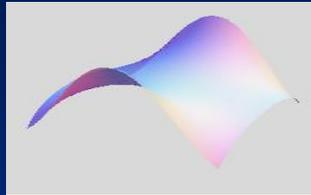
(d) $\text{Im}[y_n(x)]$

Panels (a,b) plot the real and imaginary parts of spherical Bessel functions j_n . Panels (c,d) likewise for modified Bessel functions y_n . Co-ordinate domains are $-3 < w < 3$ and $-3 < x < 3$. Colors blue, red, yellow and green label orders $n=0,1,2,3$.

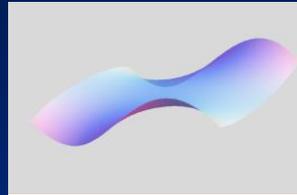
Bessel Functions for Force-Free Eigenstates

Hydrodynamics and Electromagnetics share 3D potential solutions

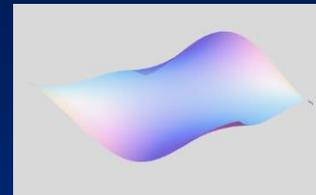
Spherical Bessel functions, order $\{0, 1, 2, 3\}$ in complex plane.



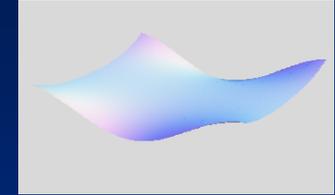
$j_0(w+ix)$



$j_1(w+ix)$

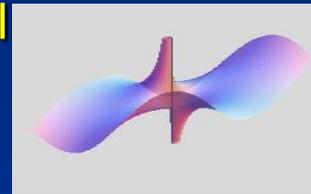


$j_2(w+ix)$

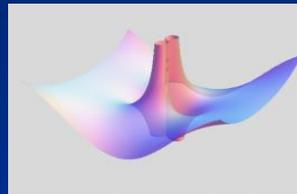


$j_3(w+ix)$

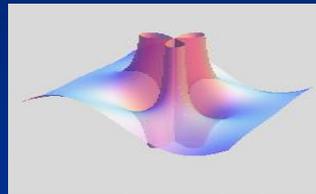
Modified spherical functions, order $\{0, 1, 2, 3\}$ in complex plane.



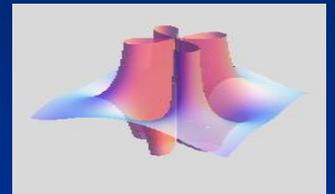
$y_0(w+ix)$



$y_1(w+ix)$



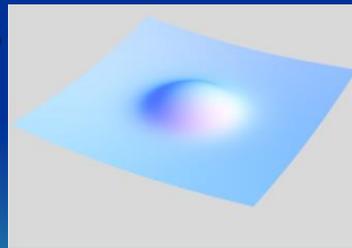
$y_2(w+ix)$



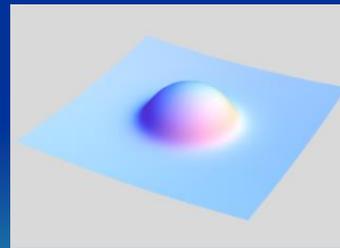
$y_3(w+ix)$

Quaternion Bessel function plots evolution of zero potential surface

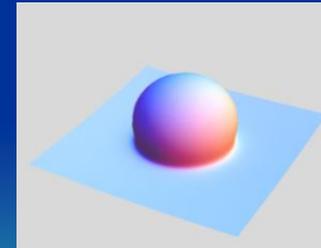
Successive panels plot 3D electric potential Φ as Bessel function



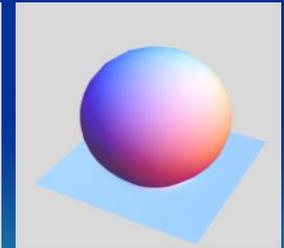
$y_0(0,x,y,1)$



$y_0(0,x,y,0.7)$



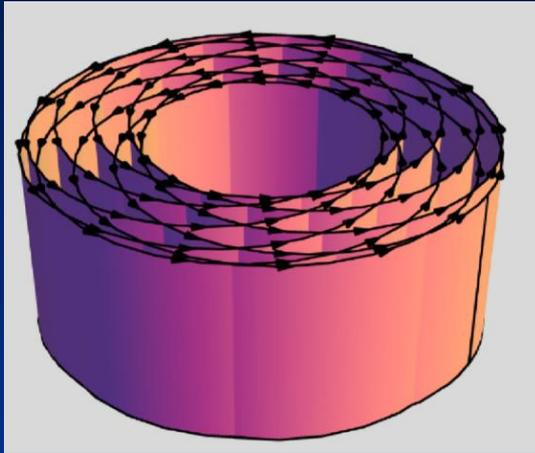
$y_0(0,x,y,0.4)$



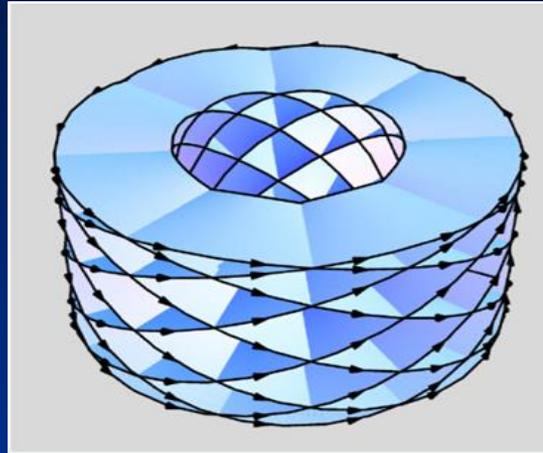
$y_0(0,x,y,0.1)$

$y_0(w,x,y,z):$

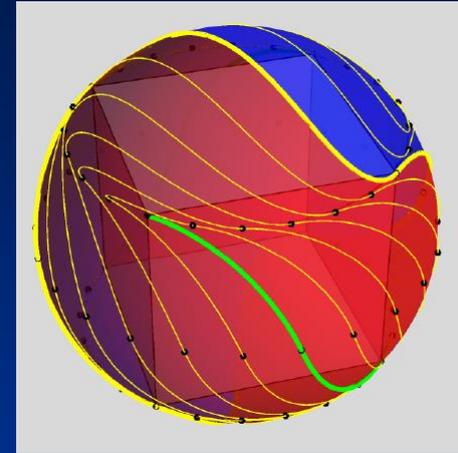
Coil Design and Manufacture



Axial channels



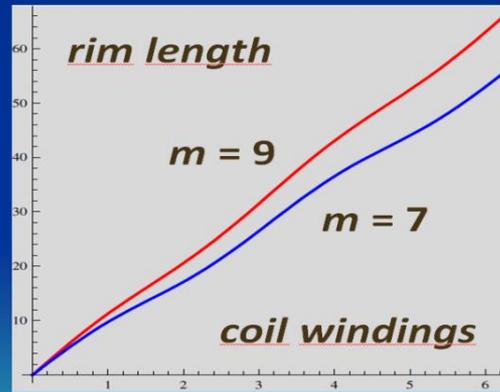
Radial channels



Nested flux loops



Machine winds litz wire



Coil properties



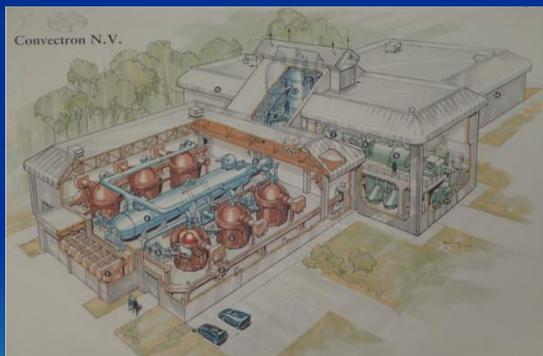
Coil delivery

Results and Conclusions

Our quaternion model of force-free ball lightning plasma obtains:

- radial electric field distributions for self-confinement,
- formation from corona discharge on aircraft window in thunderstorm,
- reversible energy storage in bosonic flux loops,
- particle acceleration into nuclear regime by vortex stretching.

Present challenge for atmospheric science: can the nuclear regime become steady?



Future fusion power plant,
< artist conception Rudolf Das,
Convectron Rotterdam 1985.
Poster session at EGU General
Assembly, Vienna 2019. >

